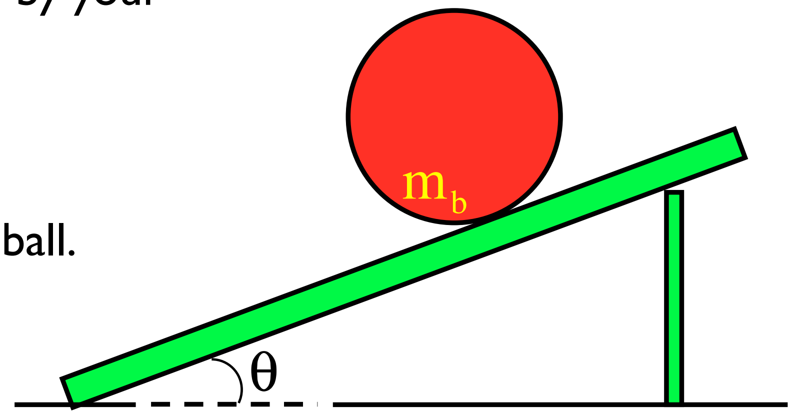
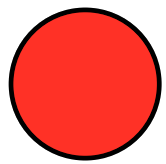
3.) A thin skinned ball sits on an incline held stationary by your finger (ah, that finger again). What is known is :



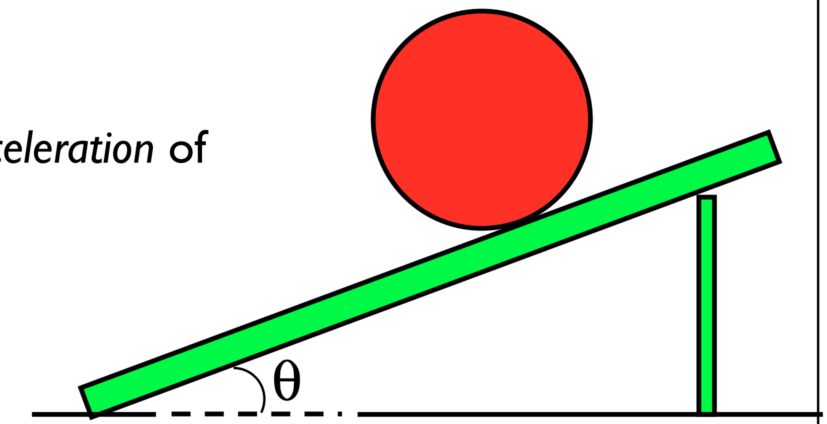
$$m,\ R,\ g,\ \theta, \text{ and } I_{\text{cm of ball}} = \frac{2}{3}mR^2$$

a.) Draw a f.b.d. identifying all the forces acting on the ball.

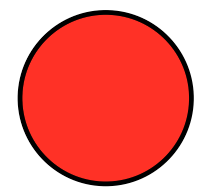


b.) Use the Parallel Axis Theorem to determine the *moment of inertia* about the *point of contact* between the ball and the incline.

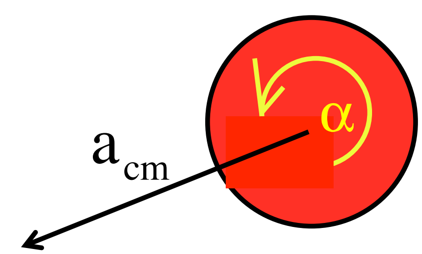
$$m,\ R,\ g,\ \theta, \text{ and } I_{\text{cm of ball}} = \frac{2}{3}mR^2$$



c.) Derive an expression for the magnitude of the *acceleration* of the ball's *center of mass*?

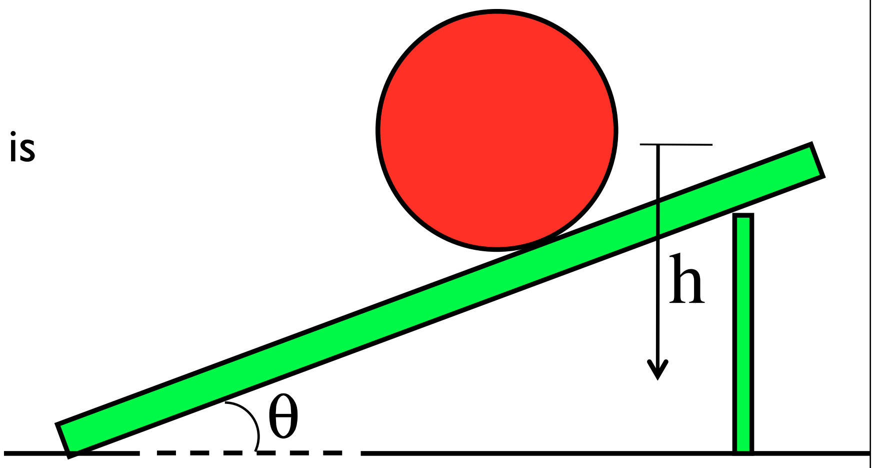


d.) What is the ball's *angular acceleration* about its *center of mass*?



$$m,\ R,\ g,\ \theta,\ \text{and } I_{\text{cm of ball}} = \frac{2}{3} mR^2$$

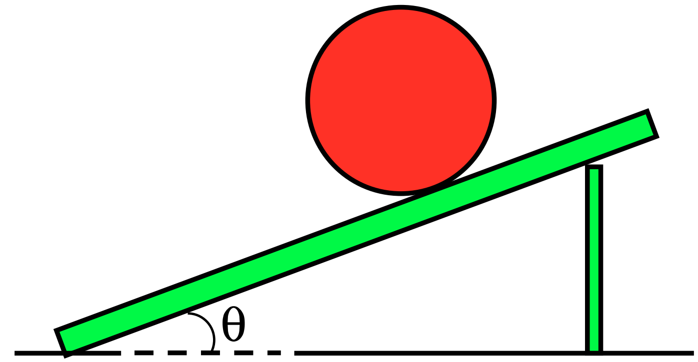
e.) The ball drops a distance "h" from rest. What is the magnitude of the *velocity* of its *center of mass*?



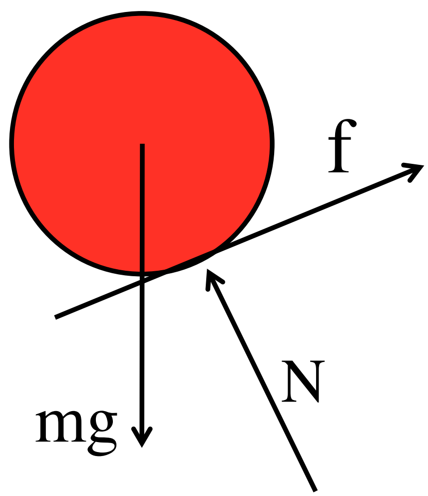
f.) After dropping "h," what is the ball's *angular velocity*?

g.) What is the *angular momentum* of the ball after dropping "h?"

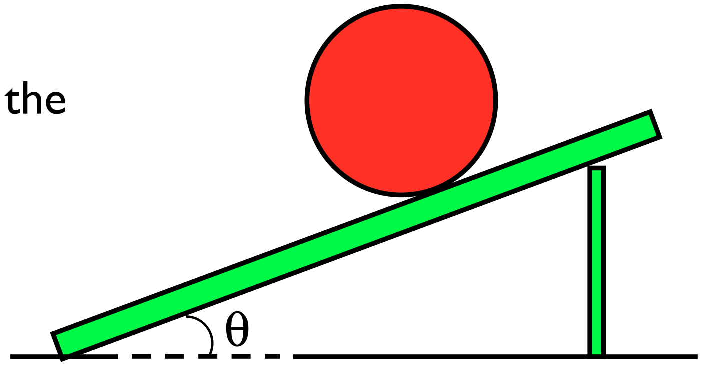
SOLUTIONS:

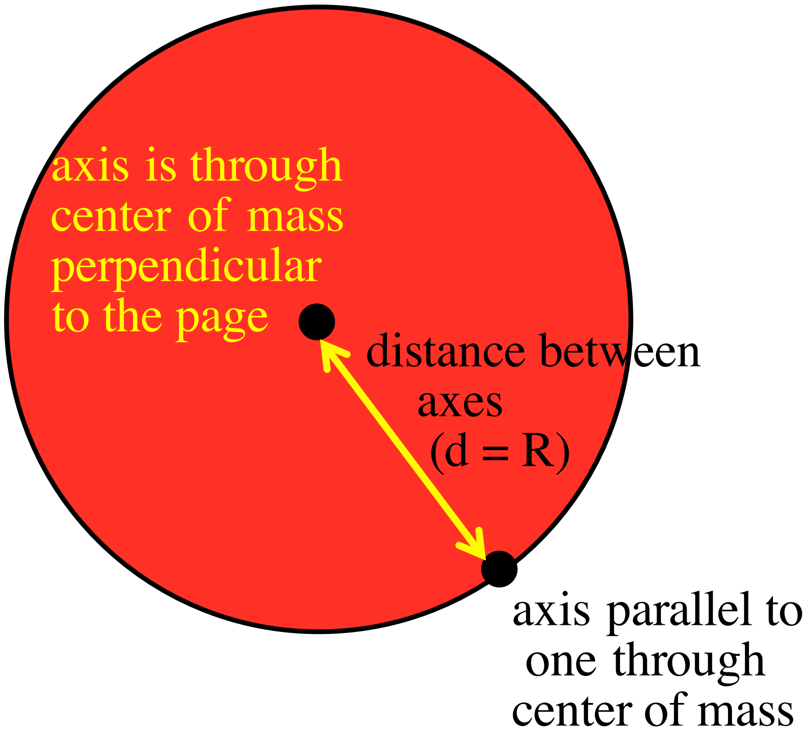


a.) Draw a f.b.d. identifying all the forces acting on the ball.



b.) Use the Parallel Axis Theorem to determine the *moment of inertia* about the point of contact between the ball and the incline.



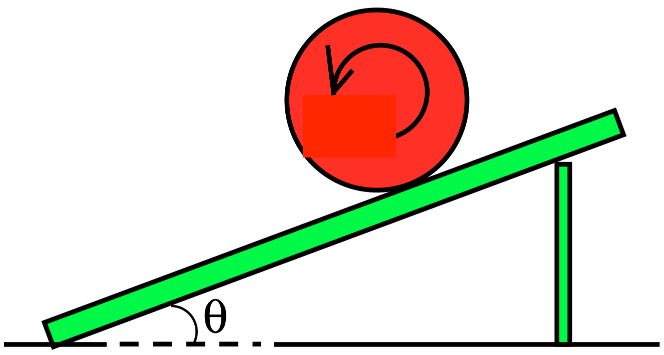


$$I_{contact\ pt} = I_{cm} + md^2$$

$$= \frac{2}{3}mR^2 + mR^2$$

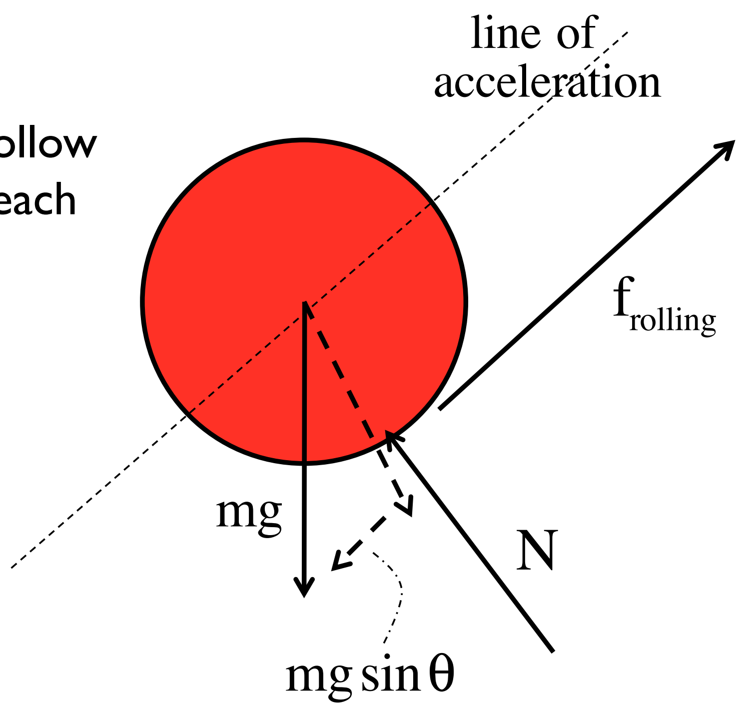
$$= \frac{5}{3}mR^2$$

c.) What is the *acceleration* of the ball's *center of mass*?



In *pinned beam* problems, you are really looking at an object that executes a *pure rotation* around a fixed point. In the case of something rolling, you are really looking at an object that has its center of mass executing *translational acceleration* AND has its mass *angular accelerating* *AROUND* the center of mass. In that case, we really want to do the evaluation relative to the *center of mass*. In other words, you will have two sources of equations: summing forces and summing torques.

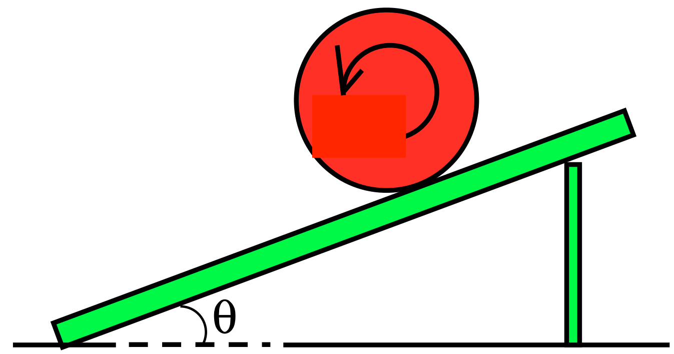
So looking at the translational version of N.S.L., we follow the steps starting with a f.b.d. (and making sure that each force is placed *where it acts*) and summing the forces, writing:



$$\sum F_x :$$

$$f_{rolling} - mg\sin\theta = -ma_{cm}$$

This yields two unknowns (f and a). To get another equation, we can *sum the torques* about *center of mass*:



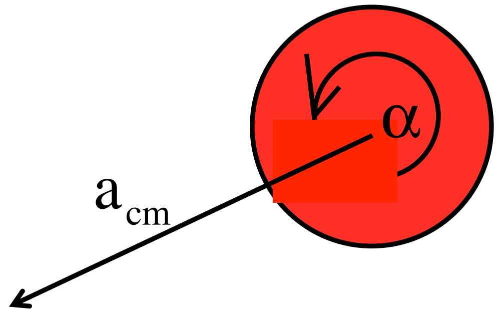
$$\sum \Gamma_{cm}: \quad \cancelto{0}{\Gamma_N} + \Gamma_f + \cancelto{0}{\Gamma_{mg}} = I_{cm}\alpha$$

$$\Rightarrow \quad f_{rolling}(R) = I_{cm}\alpha$$

$$\Rightarrow \quad f_{rolling} = \frac{\left(\frac{2}{3}mR^2\right)}{R}\alpha$$

$$\Rightarrow \quad f_{rolling} = \left(\frac{2}{3}mR\right)\alpha$$

With the angular acceleration term, we now have three unknowns. To get our final equation, we need to remember that the relationship between the *angular acceleration* about the center of mass and the *acceleration* of the center of mass is:



$$a_{cm} = R\alpha$$

So here are our relationships:

$$1.)\ a_{cm} = R\alpha$$

$$2.)\ mg\sin\theta - f_{rolling} = ma_{cm}$$

$$3.)\ f_{rolling} = \left(\frac{2}{3}mR\right)\alpha$$

If we solve for alpha in 1 and substitute that expression into 3, we get:

$$3'.)\quad f_{rolling} = \left(\frac{2}{3}mR\right)\left(\frac{a_{cm}}{R}\right)$$

If we substitute the modified 3 (i.e., 3') into 2, we get:

$$-mg\sin\theta + f_{rolling} = -ma_{cm}$$

$$\Rightarrow mg\sin\theta - \left(\frac{2}{3}m\cancel{R}\right)\left(\frac{a_{cm}}{\cancel{R}}\right) = ma_{cm}$$

$$\Rightarrow g\sin\theta = a_{cm} + \left(\frac{2}{3}\right)a_{cm}$$

$$\Rightarrow a_{cm} = \frac{3}{5}g\sin\theta$$

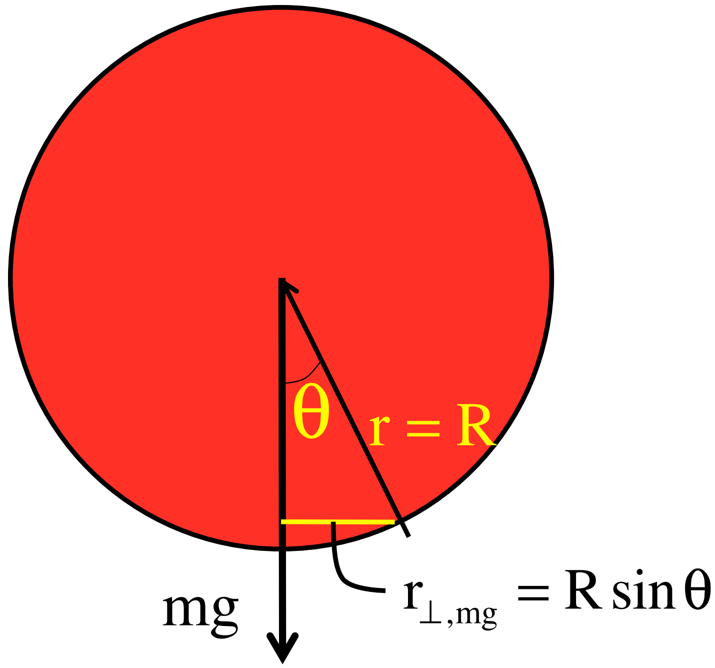
d.) What is the ball's *angular acceleration* about its *center of mass*?

Once you get here, it's gravy. All you have to do is invoke the sacred:

$$a_{cm} = R\alpha$$

Note about Part c:

Knowing the *moment of inertia* about I_p the contact point (from the Parallel Axis Theorem), there is actually an easier way to do this. Summing the torques about the point of contact with the table eliminates the need to deal with the 'f' and 'N' forces as their torques about that point will be zero. That summation looks like:



$$\sum \Gamma_{\text{contact point}}:$$

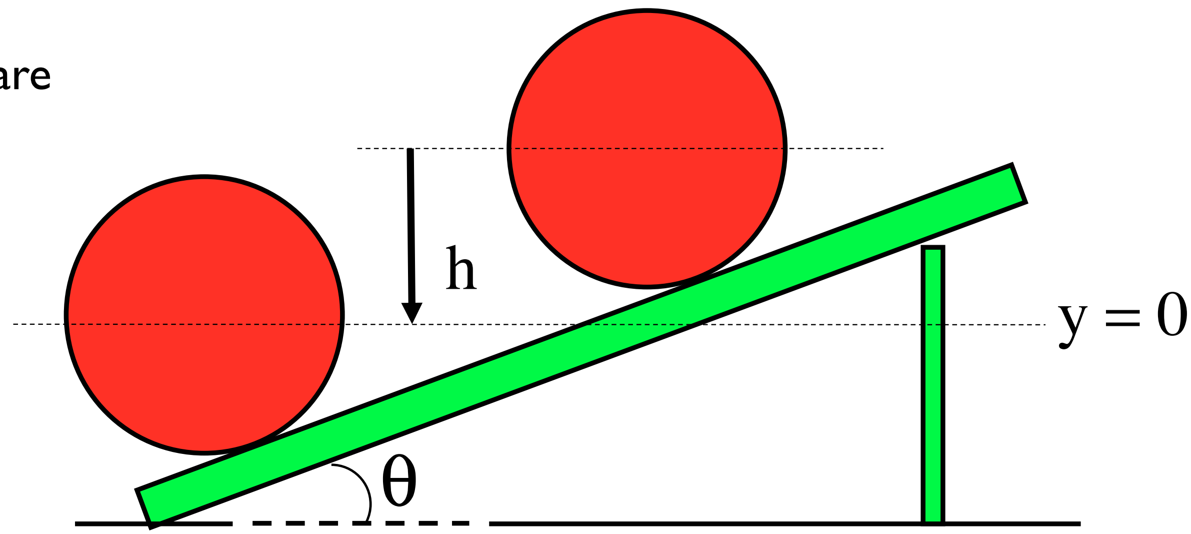
$$F_{mg} \quad r_{mg,\perp} = \left(I_p\right) \quad \alpha$$

$$mg\left(R\sin\theta\right) = \left(\frac{5}{3}mR^2\right)\left(\frac{a}{R}\right)$$

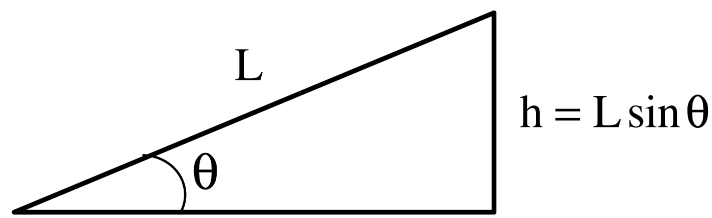
$$\Rightarrow \quad a = \frac{3}{5}g\sin\theta$$

e.) The ball drops a distance "h" from rest. What is the magnitude of its *center of mass* velocity if it loses .2 joules during the roll?

Remembering that $\omega = \frac{v}{R}$, and remembering that we are tracking the ball's *center of mass*, the situation is as shown to the right:



Minor Note: You could have been given the actual distance the ball traveled (i.e., L in the sketch to the right) instead of the ball's vertical drop "h." In that case, you would have had to use the triangle shown to determine "h" (for the gravitational potential energy quantity mgy).



Starting with the standard *conservation of energy* template and go from there:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mgh + (-.2\ J) = \left(\frac{1}{2}mv^2 + \frac{1}{2} I_{cm} \omega^2\right) + 0$$

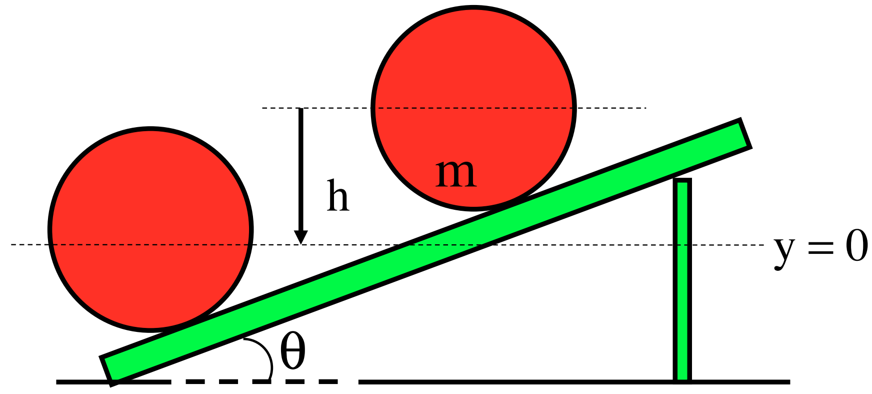
$$0 + mgh + (-.2\ J) = \left(\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2\right) + 0$$

$$\Rightarrow mgh + (-.2\ J) = \left(\frac{1}{2}mv^2 + \frac{1}{3}mv^2\right)$$

$$\Rightarrow (mgh + (-.2\ J)) = \left(\frac{5}{6}mv^2\right)$$

$$\Rightarrow v_2 = \sqrt{\frac{6}{5m}\left[mgh + (-.2\ J)\right]}$$

Minor Note: You could have approached the *kinetic energy* part at the second point as though the ball was executing a pure rotation about the contact point (remember, the contact point is instantaneously stationary). The kinetic energy of a pure rotation is:



$$KE_2 = \frac{1}{2} I_p \omega_2^2$$

where I_p is the moment of inertia about the contact point. We determine that using the parallel axis theorem in Part c (it was $I_{\text{contact pt}} = \frac{5}{3} mR^2$). This makes the problem look like (next page):

The math yields:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mgh + (-.2\ J) = \left(\frac{1}{2} I_p \omega^2\right) + 0$$

$$0 + mgh + (-.2\ J) = \left(\frac{1}{2}\left(\frac{5}{3} mR^2\right)\left(\frac{v}{R}\right)^2\right) + 0$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{6}{5m}\left[mgh + (-.2\ J)\right]}$$

IDENTICAL RESULT!

f.) After dropping "h," what is the ball's *angular velocity*?

$$v_2 = r\omega_2 \quad \Rightarrow \quad \omega_2 = \frac{v_2}{R}$$

g.) What is the *angular momentum* of the ball after dropping "h?"

$$L_2 = I_{cm}\omega_2$$